A self-similar scaling for cross-shelf exchange driven by transient rip currents

Sutara H. Suanda and Falk Feddersen

1 Scripps Institution of Oceanography, La Jolla, California, USA

Abstract

Transient rip currents, episodic offshore flows from the surf zone to the inner shelf, present a recreational beach hazard and exchange material across the nearshore ocean. The magnitude and offshore extent of transient rip-current-induced exchange and its relative importance to other inner shelf exchange processes are poorly understood. Here 120 model simulations with random, normally incident, directionally spread waves spanning a range of beach slopes and wave conditions show that the transient rip current exchange velocity is self-similar. The nondimensional exchange velocity, surf zone flushing time, and cross-shore decay length scale are scaled by beach slope and wave properties, depending strongly on wave directional spread. Transient rip-current-driven exchange can be compared to other cross-shelf exchange processes. For example, transient rip-current-driven exchange is stronger than wave-induced Stokes-drift-driven exchange up to six surf zone widths from shore.

1. Introduction

The nearshore, the \(\approx 1\) km of ocean adjacent to the coastline, consists of the surf zone, where circulation is forced by breaking surface waves, and the inner shelf immediately offshore, where flows are driven by winds, buoyancy, tides, and nonbreaking waves [Dalrymple et al., 2011; Lentz and Fewings, 2012]. Understanding cross-shelf exchange between the surf zone and inner shelf is challenging due to these dynamical differences yet essential to many areas of societal concern such as maintaining beaches, pollution dispersal, and managing coastal ecosystems. For example, tracers (e.g., terrestrial pollutants), introduced at the shoreline, are diluted in the surf zone by exchange with the inner shelf [e.g., Hally-Rosendahl et al., 2014]. Additionally, the recruitment of intertidal invertebrate larvae depends on cross-shelf exchange processes [e.g., Shanks and Wright, 1987; Fujimura et al., 2014].

A variety of physical processes can induce cross-shelf exchange. On the inner shelf, the vertical mismatch between offshore-directed undertow and onshore-wave-driven Stokes drift results in cross-shelf exchange [Lentz et al., 2008; Kirincich et al., 2009; McPhee-Shaw et al., 2011]. At subtidal time scales, cross-shelf winds can induce cross-inner-shelf exchange particularly relative to along-shelf winds [Fewings et al., 2008]. Shoreward propagating nonlinear internal waves [e.g., Pineda, 1991; Sinnett and Feddersen, 2014] and shelf eddies [e.g., Romero et al., 2013; Uchiyama et al., 2014] can also exchange material across the inner shelf. Rip currents, concentrated, offshore-directed flows from the surf zone to the inner shelf, exchange sediments [Shepard et al., 1941], pollutants [Boehm et al., 2005], and larvae [Shanks et al., 2010]. Rip currents are also a beach hazard accounting for 80% of U.S. lifeguard rescues, with more than 100 swimmer drownings each year [MacMahan et al., 2006, 2010]. Rip currents are classified as bathymetrically controlled or transient. Bathymetrically controlled rip currents occur at fixed alongshore locations near structures or on embayed or rip-channeled beaches and can be considered part of the circulation [MacMahan et al., 2010; Dalrymple et al., 2011]. Modeling [Reniers et al., 2010; Castelle and Coco, 2013] and observations [MacMahan et al., 2010; Brown et al., 2015] indicate that bathymetrically controlled rips can exchange material two to four surf zone widths from the shoreline.

In contrast, transient rip currents occur on alongshore uniform bathymetry and originate from surf zone eddies generated by finite-crest length wave breaking which then coalesce to larger scales [Peregrine, 1998; Johnson and Pattiaratchi, 2006; Spydell and Feddersen, 2009; Clark et al., 2012]. Transient rip currents occur episodically for \(O\) (1 min), have short (10–100 m) alongshore length scales [Hally-Rosendahl et al., 2014], and no preferred alongshore location [Dalrymple et al., 2011] (Figure 1a). Thus, transient rip currents can be...
Figure 1. (a) Aerial photograph of an alongshore uniform beach during a shoreline dye release [Hally-Rosendahl et al., 2014]. Dye is ejected from the surf zone to the inner shelf in plumes associated with transient rip currents. Results from a funwaveC simulation with $\beta = 0.04$ and incident $H_s = 1.1$ m, $T_p = 14$ s, $\sigma_\theta = 10^\circ$: (b) Snapshot of vertical vorticity ($\omega$) and current vectors for a subset of the model domain as a function of cross ($x$) and alongshore ($y$) coordinates for a well-developed transient rip current exiting the surf zone. (c) Significant wave height $H_s$, (d) time-mean transient rip-current-driven exchange velocity $U_{exc}$, and (e) planar bathymetry $h$ versus $x$ with shoreline at $x = 0$ m and the vertical dashed black line divides the surf zone and inner shelf ($x = L_{sz}$). The exchange velocity fit parameters ($U_{exc0}$, $x_o$, $L_d$) defined in (2) are noted in Figure 1d.

considered two-dimensional turbulence as their length scales are much longer than the water depth [Feddersen, 2014]. Although transient rip currents are ubiquitous on alongshore uniform beaches, few observational studies have attempted to quantify transient rip current exchange due to their episodic nature [Johnson and Pattiaratchi, 2004; Hally-Rosendahl et al., 2014].

Previously, surf zone eddy intensity was shown to increase with wave directional spread $\sigma_\theta$ for the same significant wave height $H_s$ and peak period $T_p$ [Spydell and Feddersen, 2009]. As transient rip currents result from surf zone eddies, these wave parameters should also determine the exchange velocity from the surf zone to inner shelf representative of rip currents. Here cross-shelf exchange by transient rip currents is investigated with 120 simulations using a wave-resolving model funwaveC, spanning a range of normally incident wave conditions and beach slopes. The model setup and methods for estimating transient rip current exchange are presented (section 2). The cross-shore profile of transient rip current exchange is self-similar and is scaled by the incident wave conditions, particularly wave directional spread (section 3). The implications of transient rip
 currents on surf zone residence time and the relative importance of transient rip-current-driven exchange to wave-driven Stokes drift exchange on the inner shelf are discussed in section 4. The results are summarized in section 5.

2. Model and Methods

The wave-resolving model funwaveC [e.g., Feddersen, 2007; Feddersen et al, 2011; Guza and Feddersen, 2012] solves the (vertically integrated) Boussinesq mass and momentum equations with nonlinear and dispersive effects [Nwogu, 1993] and parameterized wave breaking [Kennedy et al, 2000]. The model has been shown to simulate well nearshore eddy variability and dye and drifter dispersion in comparison to field observations from multiple experiments on a range of beaches [Spydell and Feddersen, 2009; Feddersen et al, 2011; Clark et al, 2011; Feddersen, 2014]. In all simulations, the bathymetry is alongshore uniform (domain length, \( L_y = 1200 \) m), with an offshore flat region (depth, \( h = 9 \) m) where waves are generated and a planar slope region which extends above the mean water line allowing wave runup. A source function method [Wei et al, 1999] generates normally incident random waves from a Pierson-Moskovitz spectrum [Pierson and Moskovitz, 1964], characterized by significant wave height \( H_s \) and peak period \( T_p \). The normally incident waves have directional spread \( \sigma_\theta \) [Kuik et al, 1988] with a Gaussian shape that is uniform at all frequencies.

A total of 120 model simulations are performed spanning a range of beach slopes (\( \beta = 0.02, 0.03, 0.04, 0.05, \) and 0.06) and wave parameters significant wave height (\( H_s = 0.5, 0.8, \) and 1.1 m), peak period (\( T_p = 8 \) and 14 s), and wave directional spread (\( \sigma_\theta = 2.5^\circ, 5^\circ, 10^\circ, \) and 20\(^\circ\)). The peak period variation includes typical sea (\( T_p = 8 \) s) and swell (\( T_p = 14 \) s) cases. This dimensional parameter space yielded substantial variation in the relevant nondimensional parameters: deep-water wave steepness \( \gamma_s \) and Irribarren number \( \text{Ir} = \beta / \gamma_s \) (see section 3). Simulations were run for 8000 s, with the last 6000 s used for analysis once mean square vorticity has equilibrated [Feddersen, 2014]. Standard analyses [Kuik et al, 1988] estimate \( H_s(x) \) and bulk \( \sigma_\theta(x) \).

Model horizontal velocities are decomposed into rotational (eddies and rip currents) and irrotational (wave) components [e.g., Spydell and Feddersen, 2009]. The net cross-shelf exchange due to transient rip currents is quantified using a rip current exchange velocity \( U_{ex}^r(x) \), representative of the time- and alongshore-averaged exchange from the offshore-directed (negative) component of the rotational flow \( u_{rot}^r \), i.e.,

\[
U_{ex}^r(x) = \left\langle \frac{1}{L_y} \int u_{rot}^r(x, y, t) dy \right\rangle. \tag{1}
\]

where \( \langle \rangle \) is a time average. The definition of \( U_{ex}^r \) (1) is analogous to the definition of estuarine total exchange flow [MacCready, 2011]. In calculating (1), the time mean of \( u_{rot}^r \) is removed to eliminate any potential standing rip current structures [i.e., Johnson and Pattiaratchi, 2006]. However, retaining the mean does not affect the results.

3. Results

An example simulation is shown in Figure 1. As random, normally incident, and directionally spread waves propagate over the bathymetry (\( h \), Figure 1e), \( H_s \) increases to a maximum, defining the breakpoint location (Figure 1c), statistically delimiting the surf zone (of width \( L_{rz} = 97 \) m) and inner shelf [e.g., Clark et al, 2010, 2011]. This \( L_{rz} \) definition gives similar results to that based on wave energy flux [e.g., Spydell et al, 2009]. Quantities located at the breakpoint are denoted by subscript “b” (e.g., \( H_{sb} \)). Onshore of the breakpoint, \( H_s \) decays toward the shoreline as finite-crest length wave breaking generates vertical vorticity (eddies) [Peregrine, 1998; Johnson and Pattiaratchi, 2006; Clark et al, 2012]. These eddies coalesce to larger scales [Spydell and Feddersen, 2009; Feddersen, 2014], resulting in episodic transient rip currents [Johnson and Pattiaratchi, 2006] with a vortex dipole signature and strong velocities, here \( > 1 \) m \( s^{-1} \) at \( x = L_{rz} \) (Figure 1b). In this example, the maximum (\( U_{ex}^r = 0.06 \) m \( s^{-1} \)) occurs at \( x = L_{rz}/2 \) (Figure 1d), decays as a Gaussian offshore, and remains significant (\( > 1 \) cm \( s^{-1} \)) up to 100 m beyond \( L_{rz} \). As \( U_{ex}^r \) represents net exchange, it is far weaker than the \( O(1) \) m \( s^{-1} \) maximum flow in an individual rip current (Figure 1b).

Similar to the example (Figure 1), over a range of \( \beta \) and wave conditions, \( U_{ex}^r(x) \) profiles are approximately Gaussian with a maximum between 0.01–0.1 m \( s^{-1} \) and a cross-shore decay scale \( L_d \) of 50–250 m,
Geophysical Research Letters

Figure 2. (a) Transient rip-current-driven exchange velocity $U_{ex}$ versus cross-shore coordinate $x$ for 120 model simulations. (b) Nondimensional $U_{ex}/U_{ex0}$ versus nondimensional cross-shore coordinate $(x - x_o)/L_d$ using the exchange velocity fit parameters ($U_{ex0}$, $x_o$, $L_d$). Nondimensional dependencies for each fit parameter: (c) normalized maximum exchange velocity $U_{ex0}/(gh_b)^{1/2}$ versus breakpoint wave directional spread $\sigma_{\theta b}$ (in degrees) with breakpoint wave steepness $S_b$ colored. (d) Location of maximum exchange velocity normalized by surf zone width $x_o/L_{sz}$ versus inverse wave steepness $S_b^{-1}$, and (e) surf-zone-width-normalized cross-shore decay length scale $L_d/L_{sz}$ versus Ir$^\infty\sigma_{\theta b}^{-1/4}$ (where $\sigma_{\theta b}$ is in radians) with $S_\infty$ colored.

Comparable to the surf zone width $L_{sz}$ (Figure 2a). The $U_{ex}'$ cross-shore profiles are well fit (regression skill > 0.8) to a Gaussian form,

$$U_{ex}(x) = U_{ex0} \exp \left[ \frac{-(x-x_o)^2}{L_d^2} \right]$$

yielding three $U_{ex}$ fit parameters ($U_{ex0}$, $x_o$, $L_d$) which are best fit using iterative least squares for each simulation. The nondimensionalized $U_{ex}/U_{ex0}$ versus $(x - x_o)/L_d$ profiles collapse into a self-similar form (Figure 2b), suggesting a scaling law for transient rip-current-driven exchange velocity $U_{ex}'(x)$.

The three exchange parameters ($U_{ex0}$, $x_o$, $L_d$) are nondimensionalized and scaled by the nondimensional wave parameters and beach slope $\beta$ (Figures 2c–2e). Relevant nondimensional surf zone parameters are the wave steepness ($S = H_s/\lambda$ where $H_s$ is the deep water $H_s$ and $\lambda$ the local wavelength given from $T_p$ and the wave phase speed) evaluated in deep water ($S_\infty = H_s/L_\infty$) or at the breakpoint ($S_b = H_{sb}/L_b$), and deep-water Irribarren number [e.g., Battjes, 1974] $Ir_{\infty, sb} = \beta/S_\infty^{1/2}$. The maximum exchange velocity $U_{ex0}$ is scaled by the breakpoint shallow water phase speed $(gh_b)^{1/2}$. The nondimensional $U_{ex0}(gh_b)^{-1/2}$ varies over a factor of 20 and depends strongly on the directional spread at the breakpoint $\sigma_{\theta b}$ with
Figure 3. Nondimensional surf zone flushing time $T_f^{1/2} (g/h_b)^{1/2} \beta$ versus breakpoint wave directional spread $\sigma_{\theta b}$ (in degrees). Red dashed curve shows $\sigma_{\theta b}$ dependence.

4. Discussion

Analogous to estuarine flushing time [MacCready, 2011; Lemagie and Lerczak, 2014], a bulk surf zone flushing time is useful in understanding surf zone to inner shelf material exchange. Here the transient rip current surf zone flushing time $T_f^r$ is defined as the time required to replace the surf zone area ($A_{sz} = h_b L_{sz}/2$),

$$T_f^r = \frac{A_{sz}}{h_b U_{ex} (x/L_{sz})} = \frac{1}{2} \frac{L_{sz}}{U_{ex} (x/L_{sz})}$$

where $U_{ex}$ is evaluated at the surf zone inner shelf boundary ($x = L_{sz}$). Short flushing times represent more rapid exchange of material. For all simulations, the dimensional $T_f^r$ generally varies between 20 min and 3 h. Based on the $U_{ex}$ scalings (equations (3a)–(3c)), the nondimensional flushing time $T_f^r (g/h_b)^{1/2} \beta$ varies more than an order of magnitude and depends proportionally on $\sigma_{\theta b}$ (Figure 3). The dimensional surf zone flushing time $T_f^r \approx (h_b/g)^{1/2}(\beta \sigma_{\theta b})^{-1}$ is thus larger for deeper breakpoint depths $h_b$, shallower surf zone slopes, and smaller directional spreads. With $\sigma_{\theta b} = 0^\circ$, no transient rip currents are generated and $T_f^r$ is infinite.
Transient rip-current-driven exchange is also estimated by advecting randomly seeded surf zone particles. The e-folding time for particles to leave the surf zone also yields a flushing time and, with (4), an exchange velocity. This velocity is well correlated with $U_{ex}^* (L_{sz}) \quad (r^2 = 0.76)$ with a regression slope of 5 due to particle recirculation typically accounted for by an exchange factor [Lemagie and Lerczak, 2014]. Thus, $U_{ex}^* (L_{sz})$ is representative of the surf zone flushing rate and the scalings (3) allow the surf zone flushing time to be quantified via (4).

On alongshore uniform coasts, wave-driven exchange across the inner shelf is usually attributed to the imbalance between vertical profiles of onshore wave-driven Stokes drift transport and offshore-directed Eulerian return flow [Monismith and Fong, 2004; Lentz et al., 2008; Lentz and Fewings, 2012], defining a Stokes exchange velocity $U_{ex}^*$. The depth-integrated model funwaveC does not resolve the vertical structure of velocity. Instead, as inner shelf Eulerian return flow (undertow) is observed to be largely depth uniform with weak vertical shear [Faria et al., 2000], Stokes-drift-induced exchange velocity $U_{ex}^*$ is estimated assuming onshore Stokes drift balanced by a depth-uniform offshore flow [Hally-Rosendahl et al., 2014],

$$U_{ex}^* = \frac{1}{8} (H_k c)^2 \int_0^{z_c} \left[ \frac{1}{h} \left( \cosh(2k_p(z + h)) - \frac{\sinh(2k_p h)}{2k_p h} \right) \right] \, dz. \quad (5)$$

where $k_p$ is the peak wave number, $c$ is the linear phase speed, and the integral is from the integrand zero crossing ($z_c$) to the surface. Note, this $U_{ex}^*$ (5) will be an overestimate if offshore Eulerian return flow is surface intensified [Putrevu and Svendsen, 1993; Lentz et al., 2008].

Previously, inner shelf drifter velocities near the surf zone have been attributed to Stokes drift [Ohlmann et al., 2012]. Here transient rip current ($U_{ex}^*$) and Stokes-drift-driven ($U_{ex}^*$) exchange velocities are compared to determine the offshore extent of transient rip current importance relative to Stokes drift. For the simulation shown in Figure 1, at the break point ($L_{sz} = 97$ m), $U_{ex}^* \approx 0.04$ m s$^{-1}$, much larger than $U_{ex}^* \approx 0.002$ m s$^{-1}$. Further offshore $U_{ex}^*$ decays more rapidly than $U_{ex}^*$, where they become equivalent at $X_0 = 353$ m with magnitude $8 \times 10^{-4}$ m s$^{-1}$ (Figure 4a). The nondimensional location (relative to the shoreline) where $U_{ex}^* = U_{ex}^*$ (defined as $X_0 / L_{sz}$) generally varies from two to six and is linearly scaled with the breaking Irribarren number $I_{br} = \beta / S_{sz}^{1/2}$ (Figure 4b). This indicates that transient rip-current-driven exchange is larger than the Stokes exchange up to two to six surf zone widths from shore well onto the inner shelf, in regions previously not thought to be influenced by the surf zone.

5. Summary

The wave-resolving model funwaveC, was used to simulate the cross-shore exchange induced by transient rip currents for a variety of normally incident, directionally spread random waves and beach slopes. The
cross-shore profile of transient rip current exchange velocity is self-similar whose maximum magnitude, peak location, and cross-shore decay length scale is accurately scaled by the beach slope and incident wave conditions. The wave directional spread strongly influences the exchange velocity due to its role in surf zone vorticity generation. The transient rip current exchange velocity cross-shore decay length scale is up to 2.5 times the surf zone width, indicating the importance of surf zone processes on the inner shelf. These scalings can be used to quantify the surf zone flushing time which also depends on wave directional spread. These scaling laws allow estimation of the transient rip current cross-shelf exchange velocity and comparison to other inner shelf exchange processes such as shoaling internal waves, shelf eddies, or wind-driven circulation. For example, transient rip current exchange velocity can be stronger than wave-induced Stokes drift exchange velocity up to six surf zone widths from shore well onto the inner shelf.

Acknowledgments
Support was provided by the National Science Foundation (NSF) and the Office of Naval Research (ONR). We thank N. Kumar, M. S. Spydell, M. H. Martinez, and R. T. Guza for helpful discussions on the manuscript. The numerical model, funwaveC, is available online at http://iod.ucsd.edu/~falk/funwaveC.html. Results from simulations used in this work are available through the corresponding author in accordance with AGU data policy.

The Editor thanks two anonymous reviewers for their assistance in evaluating this paper.

References


Erratum

In the originally published version of this article, there were errors in equation 3a and equation 3c due to a conversion error from degrees to radians. Equations 3a and 3c have since been corrected, and this version may be considered the authoritative version of record. In equation 3a, “8.7 × 10^-6” has been changed to “2.9 × 10^-2”, and in equation 3c, “4.37Ir” has been changed to “0.58Ir".