The Generation of Surfzone Eddies in a Strong Alongshore Current

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March 7, 2013

ABSTRACT

The surfzone contains energetic two-dimensional horizontal eddies with length-scales larger than the water depth. Yet, the dominant generation mechanism is not understood. The wave-resolving model funwaveC is used to simulate surfzone eddies from 4 cases examples from the SandyDuck field experiment which had alongshore uniform bathymetry. The funwaveC model is initialized with the observed bathymetry and the incident wave field in 8-m depth, and reproduces the observed cross-shore structure of significant wave height and mean alongshore current. Within the surfzone, the funwaveC modeled $E(f, k_y)$ spectra and the bulk (frequency and $k_y$ integrated) rotational velocities are consistent with the observations, demonstrating that funwaveC can be used to diagnose surfzone eddy dynamics. In the mean-squared perturbation vorticity budget, the breaking-wave vorticity forcing term is generally orders of magnitude larger than the shear-instability generation term. Thus, the shear instability mechanism is usually negligible in generating surfzone eddies, with possible exceptions for very narrow-banded in frequency and direction and highly obliquely large incident waves. The alongshore wavenumber spectra of vorticity and the breaking wave vorticity forcing reveal that the vast majority ($> 80\%$) of vorticity forcing occurs at alongshore scales $< 20$ m with a resulting red vorticity spectrum. This indicates that short-crested wave breaking dominates over wave group forcing in generating surfzone eddies and that eddy energy is cascaded to longer length-scales as in two-dimensional turbulence. A wave-resolving model that properly represents the short-crested wave breaking is suggested to be required to correctly model the surfzone eddy field.

1. Introduction

The surfzone is a place of energetic two-dimensional (2D) horizontal turbulent eddies with length-scales greater than the water depth. These eddies have rotational (as opposed to irrotational) velocities which are associated with vertical vorticity (hereafter termed vorticity). Recent observations and modeling indicate that absolute cross-shore diffusivity, inferred from dye-tracer on an alongshore uniform beach, is related to

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the bulk (surfzone-averaged) rms horizontal rotational velocities (Clark et al. 2010, 2011). Drifter-derived
time-dependent absolute cross-shore diffusivities were consistent with stirring due to surfzone eddies with
Lagrangian (not Eulerian) time-scales of $O(100) \text{s}$ (Spydell and Feddersen 2012b). When alongshore current
shear was strong, drifter-derived alongshore diffusivities were well-predicted by a shear-dispersion theory
that includes non-zero Lagrangian time-scale (Spydell and Feddersen 2012a). Thus, 2D surfzone eddies are
responsible for dispersion and dilution of surfzone tracers on alongshore uniform beaches. On rip-channeled
(not alongshore uniform) beaches, dispersion may occur due to both 2D eddies and mean circulation features
(e.g., Johnson and Pattiaratchi 2004; Brown et al. 2009).

The most commonly considered (non-passive) surfzone tracer is sediment, but also include bubbles
(Gangfeng Ma et al. 2011) and pathogens (e.g., Rippy et al. 2013; Feng et al. 2013) that impose health risk
on bathers (e.g., Haile et al. 1999). Gametes and larvae of invertebrates that live in the beach face (such as
Donax clams) are also influenced by the stirring of 2D surfzone eddies as they traverse the surfzone. Under-
standing the processes that lead to 2D surfzone eddies is critical to improved understanding of surfzone
tracer dispersion.

Time-dependent 2D eddies within the surfzone were first identified as a low-frequency, non-dispersive
ridge in frequency ($f$) and alongshore-wavenumber ($k_y$) velocity spectra $E(f, k_y)$ outside of the gravity
wave region (Oltman-Shay et al. 1989). These motions have variability on Eulerian time-scales between
50–500 s, alongshore length-scales between 40-250 m, and $E(f, k_y)$ ridge slopes are approximately equal
to the mean alongshore current $V$ (e.g., Oltman-Shay et al. 1989; Noyes et al. 2004). The magnitude of
these eddies was also generally related to $V$ (Noyes et al. 2004). However, even during times of weak $V$,
surfzone eddies are observed. For example, on a monotonic alongshore uniform beach with $V = 0 \text{ m s}^{-1}$,
the presence of a scale-dependent relative diffusivity indicated the presence of an energetic eddy field with
scales varying from 5-50 m (Spydell et al. 2007; Spydell and Feddersen 2009). During times of weak $|V|$
(< 0.25 m s$^{-1}$), eddies (rotational velocity outside of the gravity wave region) were observed with eddy
velocities of up to 0.1-0.2 m s$^{-1}$ (MacMahan et al. 2010).

With linear stability analysis, these motions were associated with the intrinsic generation mechanism of
a shear-instability of the alongshore current (e.g., Bowen and Holman 1989; Dodd et al. 1992, and many
others), and thus dubbed “shear-waves”. Shear-instabilities of the mean alongshore current, in particular
their nonlinear equilibration, have been studied with nonlinear shallow-water equation (NSWE) with steady
wave forcing both numerically (e.g., Allen et al. 1996; Slinn et al. 1998; Özkan and Kirby 1999; Noyes et al.
2004) and analytically (Feddersen 1998). NSWE model derived $E(f, k_y)$ reproduced the overall ridge-slope
of the observed $E(f, k_y)$ (Özkan and Kirby 1999; Noyes et al. 2005). However, the model energy generally
is concentrated at lower $f$-$k_y$, is less broad than observed, and often under-predicts the overall variance
(Noyes et al. 2005).

Vorticity associated with 2D surfzone eddies also is generated through the extrinsic mechanism of short-
crested breaking-wave vorticity forcing due to along-crest variation in wave dissipation (Peregrine 1998).
Recently, changes in vorticity with the passage of individual short-crested breaking waves was observed
at 10+ m length-scales (Clark et al. 2012), consistent with the theory of Peregrine (1998). On alongshore
uniform beaches, such along-crest variation is due directional spread (Kuik et al. 1988) in the incident wave
field. Wave resolving (WR) models, such a Boussinesq models (e.g., Chen et al. 2003; Feddersen et al. 2011)
generate vorticity by this mechanism. For constant incident wave energy, the surfzone averaged rms vorticity
increases with increasing wave directional spread (Spydell and Feddersen 2009). Because the length-scale of horizontal surfzone eddies is much larger than the water depth, the dynamics of surfzone eddies likely follow those of 2D turbulence (e.g., Kraichnan and Montgomery 1980; Tabeling 2002; Boffetta and Ecke 2012). A basic principle of (both freely-decaying and forced) 2D turbulence is that eddy energy cascades to longer length-scales through nonlinear interactions. Therefore, vorticity injected at short scales of $10^+ \text{m}$ evolves to larger length-scales creating a rich wavenumber spectrum of surfzone eddies.

Surfzone vorticity can also be generated at the much larger alongshore length-scales of wave-groups, generally $O(200 \text{ m})$, by alongshore variation of radiation stress gradients associated with the slow time and slow alongshore evolution of the wave envelope (wave groups) (e.g., Reniers et al. 2004; Long and Özkan Haller 2009). On alongshore uniform beaches, wave-averaged (WA) models with wave-group forcing can generate very low frequency (VLF, $< 0.004 \text{ Hz}$) rotational motions that have alongshore length scales $\approx 200 \text{ m}$ or longer (Reniers et al. 2004). The alongshore length-scale of wave groups are related to the $k_y$ width of the incident wave field. Using an example from MacMahan et al. (2010), a deep-water bi-chromatic wave train with with $f = 0.1 \text{ Hz}$ and angle separation of $\Delta \theta = 25^\circ$ gives rise to an alongshore group scale of 200 m. WA models with wave-group forcing have been successful in reproducing VLF eddy energy on rip-channeled beaches (Reniers et al. 2007, 2009). When $V \approx 0 \text{ m s}^{-1}$ and a shear-instability is not possible, modeled surfzone VLF eddy velocities, simulated with a linear (WA) wave-group model, were correlated with observations but overpredicted by 40% (MacMahan et al. 2010). However, as 2D turbulence cascades eddy energy to longer length-scales, wave group forcing cannot generate eddies at the shorter scales that have been observed (Spydell et al. 2007).

Whether shear instabilities or vorticity generated by wave breaking is the more important term in generating surfzone eddies remains an open question. In addition, the relative importance of short-crested ($O(10) \text{ m scales}$) versus wave group ($O(200) \text{ m scales}$) vorticity forcing in generating surfzone eddies is not well understood. For an idealized surfzone with an “unstable” alongshore current, Long and Özkan Haller (2009) found that vorticity generated by shear instabilities and by wave group forcing contributed approximately equally to a surfzone averaged squared potential vorticity dynamics. However, the shorter scales of vorticity injection by individual breaking waves were not included, and the model was not compared to field observations. Observations of surfzone eddies with strong $V$ have not been directly compared to a model that includes both the intrinsic shear-instability mechanism and extrinsic wave forcing (short-crested or wave groups) mechanisms.

Here the WR Boussinesq model funwaveC is used to diagnose the relative importance of the intrinsic shear-instability mechanism and the extrinsic (short-crested or wave-group) wave breaking mechanism in generating surfzone eddies. Four case examples from the SandyDuck field experiment, where observed $E(f, k_y)$ spectra were compared to a results from a NSWE model with steady forcing Noyes et al. (2005), are simulated with funwaveC. The funwaveC model is initialized with the observed bathymetry and the incident directional wave field in 8-m water depth (Section 3). The funwaveC model predicts well the cross-shore variation in significant wave height $H_s$ and mean alongshore current $V$ (Section 4.1). For these 4 cases, within the surfzone the funwaveC modeled $E(f, k_y)$ spectra (Section 4.2) and the bulk (frequency and $k_y$ integrated) rotational velocities (Section 4.3) are consistent with the observations in contrast to the NSWE model results, demonstrating that eddy dynamics can be diagnosed with funwaveC. Vorticity dynamics are examined in Section 5. The mean-squared perturbation vorticity budget is examined to determine
the relative importance of shear-instability and breaking wave vorticity forcing in generating surfzone eddies (Section 5.1). The alongshore wavenumber spectra of breaking-wave vorticity forcing and vorticity are examined to explore the dominant forcing alongshore length-scales and the induced vorticity response (Section 5.2). The results are discussed in Section 6 and summarized in Section 7.

2. Surfzone Observations

Observations were collected as part of the SandyDuck experiment that took place in Aug-Nov 1997 at the Army Corp of Engineers Field Research Facility (FRF) in Duck NC. The observations are described in detail elsewhere (Elgar et al. 2001; Feddersen and Guza 2003; Noyes et al. 2002, 2004) and are briefly discussed here. The FRF coordinate system is used where $x$ is the cross-shore coordinate increasing offshore with the shoreline near $x = 110$ m and $y$ is the alongshore coordinate. A dense cross-shore array of co-located pressure gauges and current meters (PUV) were deployed at 11 locations on a cross-shore transect extending from the the shoreline to 5.5 m water depth to measure cross-shore wave and current transformation. In addition, five alongshore arrays (denoted A1-A5 from closest to farthest from shore) of PUV were deployed (Figure 1) to measure $E(f, k_y)$ for both cross- ($u$) and alongshore ($v$) velocities using an iterative maximum likelihood estimator (Noyes et al. 2002, 2004). A pressure sensor array in 8-m water depth provides incident wave statistics including wave spectra, mean wave-angle $\bar{\theta}(f)$, and directional spread $\sigma_\theta(f)$ (Kuik et al. 1988). Wind stress $\tau_w$ was estimated from measurements at the end of the FRF pier. Observations of surfzone eddies during the 4-month-long SandyDuck experiment are described by Noyes et al. (2004) and MacMahan et al. (2010). Using a NSWE model with steady forcing, Noyes et al. (2005) simulated four, 3-hr-long case examples, 28 Aug (0828), 1 Nov (1101), 13 Nov (1113), and 17 Nov (1117), that are also
simulated here. The bathymetry and mean circulation in the instrumented area usually was alongshore homogeneous (Feddersen and Guza 2003), although alongshore variability of $V$ at the shallowest array A1 was significant on 1101 and 1113.

3. Models

a. Wave-resolving model funwaveC

The open-source, wave-resolving Boussinesq model funwaveC has been previously used to study a variety of surfzone processes including: cross-shore tracer dispersion driven by individual bores (Feddersen 2007), surfzone drifter dispersion in a weak alongshore current (Spydell and Feddersen 2009), spectral wave transformation, mean currents, and surfzone eddies (Feddersen et al. 2011), cross-shore tracer dispersion in moderate alongshore currents (Clark et al. 2011), shoreline runup (Guza and Feddersen 2012), and net circulation cells on coral reef spur and groove formations (Rogers et al. 2013). The model is briefly described here. Additional details are found elsewhere (Feddersen et al. 2011).

The time-dependent Boussinesq funwaveC model equations of Nwogu (1993) are similar to the non-linear shallow water equations, but include higher order dispersive terms. The mass conservation equation is

$$\frac{\partial \eta}{\partial t} + \nabla \cdot [(h + \eta) \mathbf{u}] + \nabla \cdot \mathbf{M}_d = 0,$$

where $\eta$ is the instantaneous free surface elevation, $t$ is time, $h$ is the still water depth, $\mathbf{u}$ is the instantaneous horizontal velocity at the reference depth $z_r = -0.531 h$, where $z = 0$ at the still water surface, and $\mathbf{M}_d$ is the dispersive term (Nwogu 1993). The two-dimensional horizontal gradient operator $\nabla$ operates on the cross-shore $x$ and alongshore $y$ directions. The momentum equation is

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -g \nabla \eta + \mathbf{F}_d + \mathbf{F}_{br} - \frac{\tau_b}{(\eta + h)} + \frac{\tau_w}{(\eta + h)} - \nu_{bih} \nabla^4 \mathbf{u}, \tag{1}$$

where $g$ is gravity, $\mathbf{F}_d$ is the dispersive term (Nwogu 1993), $\mathbf{F}_{br}$ is the breaking term, $\tau_b$ is the instantaneous bottom stress, $\tau_w$ is the surface (wind) stress. The biharmonic friction ($\nabla^4 \mathbf{u}$) term damps instabilities with hyperviscosity $\nu_{bih}$ between 0.2 to 0.3 m$^4$s$^{-1}$. The bottom stress is given by a quadratic drag law

$$\tau_b = c_d |\mathbf{u}| \mathbf{u},$$

with non-dimensional drag coefficient $c_d$ that is set between $2.3 \times 10^{-3}$-$2.6 \times 10^{-3}$ (Noyes et al. 2005). During the 4 case examples, the observed weak alongshore wind stress is applied to the model. The effect of wave breaking on the momentum equations is parameterized as a Newtonian damping (Kennedy et al. 2000) where

$$\mathbf{F}_{br} = (h + \eta)^{-1} \nabla \cdot [\nu_{br}(h + \eta) \nabla \mathbf{u}].$$

The Lynett (2006) breaking-wave eddy viscosity $\nu_{br}$ parameterization is used here with standard parameters, as in Guza and Feddersen (2012).

The alongshore uniform model bathymetry is based on those used by Noyes et al. (2005) with an additional offshore 200 m wide region of constant depth between 7–8 m depth and a sub-aerial beach extending
to 2–2.4 m above mean sea level, depending upon the case example, to allow runup. The constant depth region contains the wavemaker and a 90-m offshore sponge layer that absorbs seaward propagating waves. Shoreline runup is implemented using the “thin-layer” method (Salmon 2002), as described in Guza and Feddersen (2012). The total cross-shore domain is near 870 m for all case examples. The cross- and alongshore grid sizes are 1 m and 1.25 m, respectively. The alongshore domain width is 1500 m with alongshore periodic boundary conditions.

A wavemaker (Wei et al. 1999), located immediately onshore of the offshore sponge layer, approximately generates the target frequency-directional spectrum based upon the wave spectra, \( \bar{\theta}(f) \), and \( \sigma_\theta(f) \) observed in 8-m water depth. Full details of the model wavemaker are given in Feddersen et al. (2011). The model wavemaker is forced at many randomly-spaced discrete frequencies between 0.06 < \( f \) < 0.25 Hz at an average frequency resolution of 0.0004 Hz. The randomly spaced frequencies makes the wavemaker recurrence much longer than the model simulation. At the wavemaker, \( kh \) varied between 0.86 and 1.0 at \( \bar{f} \) (within the Nwogu 1993, limits) and \( a/h \) varied between 0.04 and 0.11 (on 11/13). The realistic modeled incident directional wave field allows for vorticity generation at the short length-scales of individual short-crested breaking waves and the longer wave group scales.

For each case example, the model was run for 8000 s and model output is analyzed over the last 5000 s. The 3000 s allowed for model spinup was sufficient for mean-square vorticity to equilibrate similar to other surfzone simulations (Feddersen et al. 2011). Modeled frequency-dependent wave spectral quantities and “bulk” sea-swell band wave quantities such as significant wave height \( H_s \) are calculated with the same estimation methods as the field observations (Section 2). The mean alongshore current \( V \) is the time-averaged \( v \).

b. Nonlinear Shallow Water Equation (NSWE) Model

The time-dependent, rigid-lid nonlinear shallow water equation model (NSWE) with steady alongshore forcing used by Noyes et al. (2005) also is described briefly here. This model is similar to those used previously (e.g., Allen et al. 1996; Slinn et al. 1998). The model rigid-lid continuity equation is \( \nabla \cdot (hu) = 0 \), and the momentum equation is similar to (1) with \( F_d \) and \( F_{br} \) set to zero. This model averages over incident wave timescales and does not include wave current interaction. The steady alongshore wave forcing is given by \( F_y = -\rho^{-1}S_{xy}/dx \), where \( S_{xy} \) is derived from a wave and roller (see Ruessink et al. 2001, for details) transformation model that best-fits the wave observations (Noyes et al. 2005). The alongshore domain width was either 1000 m or 1500 m with alongshore periodic boundary conditions. The model grid spacing was 2.5 m in both \( x \) and \( y \). Full details can be found in Noyes et al. (2005).

4. Results: Model-data comparison

a. Significant wave height \( H_s \) and mean alongshore current \( V \)

As a precondition to testing the model’s ability to accurately simulate the surfzone eddy field, model data comparison is performed for bulk parameters such as significant wave height \( H_s \) and mean alongshore current \( V \). For the four case examples, the 8-m depth incident \( H_s \) varied between 0.78 (0828) and 2.70 m (1113). On 0828, the incident (8-m depth) \( H_s = 0.78 \) m, wave breaking begins between A1 and A2, and the
model reproduces the observed cross-shore structure of \( H_s \) (Fig. 2a). The mean alongshore current \( V \) is near-zero offshore of the surfzone, with surfzone maximum \(|V| \approx 0.45 \text{ m s}^{-1}\), and the cross-shore structure is well reproduced by the model (Fig. 2b). The model similarly reproduces the observed \( H_s \) (Fig. 2d,g,j) and \( V \) on the other three days (Fig. 2e,h,k). On 1101, the larger incident \( H_s = 1.48 \text{ m} \), results in wave breaking just offshore of A2 driving a strong, narrow surfzone alongshore current jet with maximum \(|V| \approx 1 \text{ m s}^{-1}\). On 1113, the large incident \( H_s = 2.70 \text{ m s}^{-1}\) resulted in a wide surfzone that encompassed all five alongshore arrays (Fig. 2g) with broad relatively strong alongshore current (\(|V| \geq 0.5 \text{ m s}^{-1}\)) at each (Fig. 2h). On 1117, incident \( H_s = 0.96 \text{ m} \) with wave breaking just offshore of A2 (Fig. 2j), and the large incident wave angle drives drives relatively strong jet-like surfzone currents up to \(|V| \approx 0.75 \text{ m s}^{-1}\) (Fig. 2l). In contrast to the simple one-dimensional modeled \( H_s(x) \) and \( V(x) \) structure in Noyes et al. (2005), funwaveC model parameters were not tuned here to minimize model-data error.

b. Frequency-Alongshore Wavenumber Spectra \( E(f,k_y) \)

Here, the funwaveC model derived frequency-alongshore wavenumber spectra \( E(f,k_y) \) are compared to the observed and NSWE modeled \( E(f,k_y) \) reported in Noyes et al. (2005) at alongshore arrays within and seaward of the surfzone. The NSWE with steady wave forcing only generates eddies via the intrinsic shear-instability mechanism, whereas the wave-resolving funwaveC also allows eddy generation through both short-crested and wave-group breaking-wave vorticity generation mechanism.

On 0828 at the surfzone A1 location, the funwaveC \( E(f,k_y) \) is qualitatively more similar to the observations than the NSWE (Fig. 3a-c). The NSWE \( E(f,k_y) \) is a narrow ridge with energy concentrated at lower \( k_y (< 0.01 \text{ m}^{-1}) \) and \( f (< 0.004 \text{ Hz}) \), in contrast to the broader \( f \) and \( k_y \) range of \( E(f,k_y) \) (up to
$k_y = 0.024 \text{ m}^{-1}$ and $f = 0.02 \text{ Hz}$) in the funwaveC and observed. The funwaveC and observed $E(f, k_y)$ ridge is wider in frequency than the NSWE $E(f, k_y)$. At A2 (middle row of Fig. 3), located just seaward of the surfzone, NSWE energy is reduced significantly to very low $k_y$ whereas the funwaveC and observed $E(f, k_y)$ are more qualitatively consistent. At A3 (bottom row of Fig. 3), the energy is weak everywhere. As discussed by Noyes et al. (2005), the $E(f, k_y)$ ridge-slope generally is consistent with the local $V$ (solid line in Fig. 3) at the three cross-shore locations, in contrast to equilibrated weakly nonlinear shear-wave theory (Feddersen 1998).

The features in the NSWE, funwaveC, and observed qualitative $E(f, k_y)$ comparison are similar on other days. On 1101 within the surfzone at A1 and A2, the observed $E(f, k_y)$ has a broad ridge with slope similar to the local $V$ that extends to large $k_y = 0.024 \text{ m}^{-1}$ and higher $f = 0.02 \text{ Hz}$ (Fig. 4c,f). The
funwaveC modeled surfzone $E(f, k_y)$ have similar features as the observed (Fig. 4c,f). In contrast, the NSWE $E(f, k_y)$ have narrow ridges limited to $k_y \leq 0.01$ m$^{-1}$ (Fig. 4a,d). Seaward of the surfzone at A3 and A4, the observed and funwaveC $E(f, k_y)$ is limited to lower frequencies (largely $f < 0.005$ Hz) but a broad range of $k_y$ (Fig. 4h,i,k,l). The A3 and A4 NSWE $E(f, k_y)$ is also confined to lower frequencies, but not a broad range of $k_y$ (Fig. 4g,j).
These surfzone features (Fig. 3 and 4) also are clearly seen in the very wide (spanning A1-A5) surfzone of 1113 (Fig. 5). The observed $E(f, k_y)$ has a broad ridge that extends to large $k_y = 0.024 \text{ m}^{-1}$ and higher $f = 0.025 \text{ Hz}$ (Fig. 5, right column). The funwaveC $E(f, k_y)$ are consistent with the observed (Fig. 5, middle column), whereas the NSWE $E(f, k_y)$ are again narrow ridges with reduced energy at larger $k_y$ and $f$ (Fig. 5, left column). Both models and observed $E(f, k_y)$ ridge slopes are consistent with the local $V$ at all arrays.

On 1117, observed $E(f, k_y)$ could not be estimated at A1, located mid-surfzone near the maximum $V$ (Fig. 2k). Thus, the comparison is performed only at A2–A4 (Fig. 6). At the outer-surfzone A2 (Fig. 2j), the observed and funwaveC $E(f, k_y)$ are similar and have general features consistent with the other case examples. At A2, the NSWE model $E(f, k_y)$ is most qualitatively similar to the observed of all the surfzone array cases. Although the ridge is not sufficiently broad, significant NSWE energy is present at higher $k_y$ (up to $0.02 \text{ m}^{-1}$, Fig. 6a), than all other case examples. Seaward of the surfzone at A3 and A4, the observed, funwaveC, and NSWE $E(f, k_y)$ have similar features to the other seaward of the surfzone locations on the other case examples.

c. Rotational Velocities

At surfzone locations, the funwaveC modeled $E(f, k_y)$ is qualitatively far more consistent with the observed $E(f, k_y)$ than is the NSWE modeled $E(f, k_y)$ (Figs. 3-6). The comparison is now made quantitative by comparing the rotational velocities $u_{\text{rot}}$ and $v_{\text{rot}}$ associated with these $E(f, k_y)$ across all case example days within and seaward of the surfzone. Observed, NSWE modeled, and funwaveC modeled root-mean-square rotational (i.e., vortical) velocities $u_{\text{rot}}$ and $v_{\text{rot}}$ are calculated by integrating the respective $E(f, k_y)$ over the non-gravity wave region between $f = 0.00165 \text{ Hz}$ and $f = 0.25 \text{ Hz}$ and over alongshore wavenumber $k_y$ outside of the gravity wave region and taking a square root (see Noyes et al. 2004, for processing details). This procedure removes irrotational infragravity wave energy, leaving only rotational (eddy) velocity contributions.

At surfzone locations, the observed $u_{\text{rot}}$ varies between $0.08-0.21 \text{ m s}^{-1}$ and $v_{\text{rot}}$ varies between $0.07-0.16 \text{ m s}^{-1}$ (black asterisks in Fig. 7). The funwaveC model reproduces the surfzone observed $u_{\text{rot}}$ and $v_{\text{rot}}$ (Fig. 7a,b, respectively), with root-mean square errors of $(0.05, 0.04) \text{ m s}^{-1}$, respectively and small bias. The NSWE model generally underpredicts the observed $u_{\text{rot}}$ and $v_{\text{rot}}$ significantly (Fig. 7c,d) with larger rms errors $(0.09, 0.06) \text{ m s}^{-1}$ and large bias. At seaward of the surfzone locations (circles in Fig. 7), $u_{\text{rot}}$ and $v_{\text{rot}}$ are $\leq 0.04 \text{ m s}^{-1}$, much weaker than within the surfzone, and both funwaveC and NSWE model reproduce the observed $u_{\text{rot}}$ and $v_{\text{rot}}$.

5. Results: Vorticity

The ability of the funwaveC model to reproduce the observed surfzone eddy field (Section 4) means that funwaveC can be used to diagnose the dominant processes generating surfzone eddies. Vorticity $\omega$ and perturbation vorticity $\omega'$ are the natural variables to diagnose competing surfzone eddy generation mechanisms as they corresponds only the (rotational) eddies and not the (irrotational) infra-gravity wave motions.

a. Mean squared perturbation vorticity budget

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To diagnose the relative importance of shear instabilities or breaking-wave vorticity forcing to the surfzone eddy field, the mean square perturbation vorticity $\omega^2$ (where the overbar represents an average) is examined. Beginning with the shallow-water based vorticity equation, removing the mean, and multiplying by $\omega'$ and averaging yields the $\omega^2$ evolution equation (e.g., Salmon 1998),

$$\frac{1}{2} \frac{\partial \omega^2}{\partial t} + \ldots = -\omega' u' \frac{dV}{dx} + \omega \nabla \times \mathbf{F}_{br} + \ldots$$  \hspace{1cm} (2)

where the first and second terms on the right-hand side of (2) are the shear-instability and breaking-wave contributions, respectively. Other terms not shown in (2) are the advection and stretching terms that transform but do not generate eddies, and the bottom-friction induced $\omega^2$ decay. The equation for $\omega^2$ is considered here for simplicity, but the equation for mean-square perturbation potential vorticity, considered by Long and Özkan Haller (2009) who integrated across the surfzone, yields identical results, as does examining the magnitude of terms in the perturbation vorticity equation. The first and second terms on the rhs of (2) are estimated from the 1 Hz model output with the overbar representing both a time-average (over last 5000 s of model run) and an alongshore average over the domain, implying that $\phi(x, y, t) = \bar{\phi}(x)$. Thus, both rotational and irrotational motions contribute to $u'\omega'$. The results are identical if the perturbation velocity $u'$ derived from a rotational/irrotational velocity decomposition (Spydell and Feddersen 2009) is used instead.

The breaking wave vorticity forcing $(\omega' \nabla \times \mathbf{F}_{br})$ dominates over the shear-instability mechanism $(\omega' u' \frac{dV}{dx})$ at all cross-shore locations and all case examples (compare blue and green-dashed curves in Fig. 8). Within the surfzone, the breaking-wave term has magnitude 1 to 6 s$^{-3}$ across all days. On 1101 and 1113, the shear instability term is negligible compared to the breaking wave term (Fig. 8b,c). Only on 0828, near $x = 170$ m (Fig. 8a) where $\frac{dV}{dx}$ is strong (Fig. 2b), is the shear-instability term within an order of magnitude of the breaking wave term. Although the mean shear $\frac{dV}{dx}$ can be quite large (up to 0.01 s$^{-1}$), surfzone eddies in these four case examples were generated by breaking-wave vorticity forcing and not by a shear instability.

b. Alongshore length-scales of vorticity forcing and vorticity

Given that the shear-instability mechanism is negligible relative to the breaking-wave vorticity forcing in driving surfzone eddies, the remaining question is: What is the relative importance of wave group forcing (with long alongshore length-scales) or individual breaking waves (with shorter length-scales) in driving surfzone eddies.

This question is addressed by examining the cross-surfzone averaged (indicated with a hat) alongshore wavenumber spectra of the breaking-wave vorticity forcing $\hat{E}_{\nabla \times \mathbf{F}_{br}}(k_y)$ and the vorticity $\hat{E}_\omega(k_y)$. At a particular cross-shore location, alongshore wavenumber spectra of vorticity $(E_\omega(x, k_y))$ and breaking-wave vorticity forcing $(E_{\nabla \times \mathbf{F}_{br}}(x, k_y))$ are estimated by time-averaging 1 Hz alongshore periodograms over the 5000 s of model output at various cross-shore surfzone locations. These spectra are then cross-shore averaged from the shoreline to the “breakpoint” to arrive at a single surfzone averaged spectra indicated with a hat, i.e., $\hat{E}_\omega(k_y)$. To account for cross-shore variable depth inducing stretching in vorticity dynamics, the cross-shore average is weighted by $h^{-2}$, i.e.,

$$\hat{E}_\omega(k_y) = \left\langle E_\omega(x, k_y) \frac{\hat{h}^2}{h(x)^2} \right\rangle,$$  \hspace{1cm} (3)
where $\langle \rangle$ represents a cross-surfzone average, $h(x)$ is the local water depth, and $\hat{h}^2$ is the cross-surfzone mean squared depth. In effect, $\hat{E}_y$ represents the cross-surfzone averaged potential vorticity ($\omega/h$) spectrum. Whether this weighting ($\hat{h}^2/h(x)^2$) is included or not in the cross-shore average does not affect the results.

On all case examples, $\hat{E}_y \times F_{br}(k_y)$ is broad in $k_y$ (Fig. 9, left column). On 0828, 1101, and 1113, $\hat{E}_y \times F_{br}(k_y)$ is white (flat) for $k_y < 0.02$ m, has a maximum roughly twice the background value between 0.05 $< k_y < 0.12$ m$^{-1}$ (denoted by vertical arrows in Fig. 9a1,b1,d1), and decays at higher $k_y$. On 1113 with a wide surfzone, $\hat{E}_y \times F_{br}(k_y)$ is essentially white at all $k_y$. For all case examples, between 80% (0828,1101) to 90% (1117) of the $\nabla \times F_{br}$ variance is at $k_y > 0.05$ m$^{-1}$, equivalent to alongshore length-scales $< 20$ m. The vorticity forcing magnitude varies significant cross-surfzone, but the spectral shape does not (red dashed lines in Fig. 9,left). Therefore, the forcing length-scales do not vary significantly across the surfzone.

The surfzone-averaged vorticity spectrum $\hat{E}_y(k_y)$ that results from the vorticity forcing is generally red at $k_y > 10^{-2}$ m$^{-2}$ with a steep fall-off at $k_y > 10^{-1}$ m$^{-1}$ (Fig. 9,right). Although vorticity forcing is concentrated at $k_y > 0.05$ m$^{-1}$ (< 20 m length-scale), longer length-scales contribute to vorticity variance. Between 75% (1113) to 40% (0828) of the vorticity variance is at $k_y > 0.01$ m s$^{-1}$ (< 100 m scales). The red vorticity structure indicates 2D turbulence (Tabeling 2002) with nonlinear energy transfers from the higher $k_y$ (> 0.05 m$^{-1}$) vorticity forcing to lower $k_y$. Although the breaking-wave vorticity forcing magnitude varies significantly across the surfzone (dashed curves in Fig. 9,left), the cross-surfzone variation of $\hat{E}_y(k_y)$ is weak (dashed curves in Fig. 9,right), indicating that eddy properties are well stirred within the surfzone.

The detailed structure of $\hat{E}_y(k_y)$ varies across case example days with no consistent $\hat{E}_y(k_y) \propto k_y^{\gamma}$ power-law structure (indicated with dash-dot lines in Fig. 9,right). The $\hat{E}_y(k_y)$ power-law exponent $\gamma$ has a shift near the maximum in the vorticity forcing for the 3 case examples (0828, 1101, 1117) with a clear vorticity forcing maximum (Fig. 9,a,b,d). In classic 2D turbulence forced at a single $k_F$ (e.g., Kraichnan and Montgomery 1980), the velocity spectrum $E_y(k_y)$ has power-law exponent $-5/3$ at at $k < k_F$ (energy cascade regime) and power-law exponent $-3$ at $k > k_F$ (enstrophy cascade regime). Relating vorticity and velocity wavenumber spectrum by $\hat{E}_y(k_y) \propto k_y^2 \hat{E}_v(k_y)$, the $-5/3$ energy cascade power-law exponent corresponds to $\gamma = 1/3$ and the $-3$ enstrophy cascade exponent corresponds to $\gamma = -1$. Although the surfzone-averaged $\hat{E}_y(k_y)$ exhibit many characteristics of 2D turbulence, classic energy $\gamma = 1/3$ or enstrophy $\gamma = -1$ cascade regimes are not regularly exhibited (Fig. 9,right).

6. Discussion

A model must accurately simulate both the magnitude and length-scales of the surfzone eddies to properly represent surfzone tracer mixing. The qualitative similarity across all $k_y$ between funwaveC and observed $E(f,k_y)$ (Figs. 3-6) and the quantitative similarity between $u_{rot}$ and $v_{rot}$ (Fig. 7) indicates that the funwaveC model is accurately simulating the surfzone eddy field during these 4 SandyDuck case examples. This is consistent with the funwaveC model-data comparison from the HB06 experiment (Feddersen et al. 2011) which compared $u_{rot}$ and $v_{rot}$ derived from a different estimator (Lippmann et al. 1999), as $E(f,k_y)$ could not be estimated. It is also consistent with the funwaveC reproducing HB06 experiment observed tracer-derived surfzone diffusivity (Clark et al. 2011).

With a WR model, the extrinsic breaking-wave vorticity forcing was found to dominate over the intrinsic
shear-instability mechanism (Fig. 8). In contrast, with a group forced WA model for an idealized moderate $V$ surfzone example Long and Özkan Haller (2009) found that breaking wave vorticity forcing and shear instability contributed approximately equally. This difference likely is due to the WA-model not including the forcing at short-scales (particularly $< 20$ m) which dominates the forcing variance (Fig. 9,left). From the range of wave and current conditions (particularly the large $dV/dx$) considered here, one may conclude that in most natural surfzones the shear instability eddy generation mechanism is negligible relative to the breaking-wave forcing, with possible exceptions for very narrow-banded, highly obliquely, large incident waves.

Wave breaking forces vorticity predominantly at $< 20$ m scales, which through a nonlinear 2D turbulent eddy cascade creates a vorticity spectrum with energy at longer scales (smaller $k_y$). This is consistent with the $V = 0$ m s$^{-1}$ observed and modeled scale-selective (10–50 m) surfzone drifter dispersion (Spydell and Feddersen 2009). It is also consistent with eddy energy present out to $k_y = 0.05$ m$^{-1}$ in WR transient rip current modeling (Johnson and Pattiaratchi 2006). However, this interpretation contrasts with a wave-averaged (WA) VLF surfzone eddy model directly forced by wave groups when $V \approx 0$ m s$^{-1}$ (MacMahan et al. 2010). This linear wave-group model, which neglects nonlinear energy transfers generates rms surfzone eddy velocities correlated with, but 40% larger than, observed (MacMahan et al. 2010). However, except for a single case example, eddy length-scales were not generally examined. Given the WR breaking-wave vorticity forcing short ($k_y > 0.05$ m$^{-1}$) length-scales, it is unlikely that a WA model accurately simulates the surfzone eddy length-scales required for surfzone mixing. Estimating eddy $k_y$ structure with weak $V$ (no eddy propagation) may be difficult with a lagged array designed for studying waves where most separations are larger ($> 50$ m) than many surfzone eddies (Fig. 1). Future work could compare WR and WA models in both weak and strong $V$ contexts.

As 2D turbulence is not a “wave” with a distinct dispersion relationship, it is naturally studied in wavenumber space (e.g., Kraichnan and Montgomery 1980). Both Noyes et al. (2005) and Long and Özkan Haller (2009) discussed that alongshore eddy advection yields an $E(f, k_y)$ ridge slope approximately equal to the local $V$. More specifically, for an assumed “frozen” turbulence field, alongshore eddy advection induces a “Taylor hypothesis” mapping from wavenumber to frequency via $f = k_y V$, giving the appearance of a “shear-wave” dispersion-relationship (e.g., Oltman-Shay et al. 1989; Noyes et al. 2004). This explains why VLF ($f < 0.004$ Hz) energy is observed to be prevalent during weak $V$ conditions (MacMahan et al. 2010). With this interpretation, VLF energy is not dynamically distinct from infragravity-band (IG, $0.004 < f < 0.03$ Hz) rotational energy. With the same eddy field, but stronger $V$, eddy $k_y$ variability would be mapped to higher frequency than the observed VLF. The observed and funwaveC modeled $E(f, k_y)$ ridge frequency-broadening at higher $k_y$ (see A1 on 0828, Fig. 3) possibly could be explained by oscillating advection of larger $k_y$ motions by smaller $k_y$ motions result in increasing frequency spreading (e.g., Lumley and Terray 1983).

The near-white $\hat{E}_{\nabla \times F_{br}}(k_y)$ spectrum with a high $k_y$ ($> 0.1$ m$^{-1}$) fall-off (Fig 9,left) is approximately consistent with stochastic top-hat function vorticity forcing with length scale of $O(10)$ m throughout the surfzone, consistent with the results of Clark et al. (2012). This suggests that the effect of short-crested breaking-wave vorticity generation could be stochastically parameterized within a wave-averaged model, analogous to the stochastic breaking-wave forcing of the mixed layer by Sullivan et al. (2007).

That the surfzone $\hat{E}_\omega(k_y)$ does not consistently exhibit 2D turbulence classic energy or enstrophy cas-
cade regimes (Fig. 9, right) is not unexpected. The stochastic vorticity forcing is broad in wavenumber \( k_y \) space, the \( \approx 100 \)-m wide (except 1113) surfzone region within which eddies are forced imposes an additional length-scale, and the cross-shore variable depth will alter vorticity dynamics through vortex stretching. These factors all potentially lead to differences from classic 2D turbulence, and result in a complex and interesting eddy field. Future work will examine the structure of the surfzone eddy field, and how eddies transform as they leave the surfzone where the depth increases and the forcing ceases.

7. Summary

Here the wave-resolving (WR) Boussinesq model funwaveC is used to simulate 4 cases examples from the SandyDuck field experiment presented by Noyes et al. (2005). Here, the model funwaveC is initialized with the observed bathymetry and the incident wave energy, mean direction, and directional spread at each frequency. For these four cases, the funwaveC model reproduces the predicted cross-shore significant wave height \( H_s \) and mean alongshore current \( V \) (Section 4.1). The funwaveC modeled \( E(f, k_y) \) spectra (Section 4.2) and the bulk rotational velocities (Section 4.3) are consistent with the observations. This gives confidence that the model can be used to diagnose eddy dynamics. Using the mean-squared perturbation vorticity budget (Section 5.1), breaking wave vorticity forcing dominates the shear-instability mechanism. As these cases have strong \( V \) and large shear \( dV/dx \), this likely applies to most surfzones with possible exceptions for very narrow-banded in frequency and direction, highly obliquely incident waves.

The alongshore wavenumber spectra of vorticity and the breaking wave vorticity forcing reveal that the vast majority (\( > 80\% \)) of vorticity forcing occurs at alongshore scales \( < 20 \) m with a resulting red vorticity spectrum. This indicates that short-crested wave breaking dominates over wave group forcing in generating surfzone eddies and that eddy energy is cascaded to longer length-scales as in two-dimensional turbulence. Vorticity alongshore wavenumber spectra did not follow classic 2D turbulence power-law scalings due to the broad forcing, the finite cross-shore forcing region, and cross-shore variable depth. In sum, these results suggest that a wave-resolving model that properly represents the short-crested wave breaking is required to correctly model the surfzone eddy field.

Acknowledgments. This work was funded by the National Science Foundation and the Office of Naval Research. ONR funded the SandyDuck field experiment. S. Elgar, T. H. C. Herbers, R. T. Guza, and B. Raubenheimer were PIs on the PUV array component of the experiment used here. The instruments were deployed and maintained by staff from the Center for Coastal Studies, Scripps Institution of Oceanography. Staff from the U.S. Army Corps of Engineers Field Research Facility, Duck, North Carolina, provided processed survey data and data from their pressure array in 8-m water depth. The open source funwaveC model was developed by F. Feddersen is available at http://falk.ucsd.edu. Matthew Spydell provided useful feedback on the manuscript.

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Fig. 5. SandyDuck experiment (1113) cross-shore velocity alongshore-wavenumber-frequency spectra $E(f, k_c)$ for (left) NSWE modeled, (middle) funwaveC modeled, and (right) observed at cross-shore locations (top to bottom) A1–A5 which are all within the surfzone. See Fig. 3 for further details.
Fig. 6. SandyDuck experiment (1117) cross-shore velocity alongshore-wavenumber-frequency spectra $E(f, k_y)$ for (left) NSWE modeled, (middle) funwaveC modeled, and (right) observed at cross-shore locations (top) A2, (middle) A3, and (bottom) A4. See Fig. 3 for further details.
Fig. 7. Modeled versus observed ($u_{rot}$, left and $v_{rot}$, right) rotational velocities for funwaveC (top-a,b) and the NSWE (bottom-c,d) at locations within (black asterisks) and seaward (red circles) of the surfzone. The black line represents the 1:1 relationship.
Fig. 8. The breaking-wave forcing $|\mathbf{F}_{br}|$ (blue) and shear-instability $|\omega' u' dV/dx|$ (green dashed) term magnitude in the mean perturbation vorticity squared budget (2) versus FRF cross-shore coordinate $x$ for the 4 case examples: (a) 0828, (b) 1101, (c) 1113, and (d) 1117.
Fig. 9. The cross-shore mean (solid blue) and ± standard deviation (red dashed) of (left column) breaking-wave vorticity forcing $E_{\nabla \times F_{br}}$ and (right column) vorticity $E_{\omega}$ spectra versus alongshore wavenumber $k_y$ for top to bottom (a) 0828, (b) 1101, (c) 1113, and (d) 1117. The cross-shore averaging methodology is given in (3). In the left column, the vertical arrows in (a1),(b1), and (d1) indicates the wavenumber when a distinct maximum $E_{\nabla \times F_{br}}$ occurs. On 1113, there was not a distinct maximum. In the right column, the dash-dot lines indicate approximate power law regions where $E_{\omega} \propto k_y^\gamma$ where the $\gamma$ value is indicated near the dashed line. The Nyquist wavenumber is $k_y = 0.4$ m$^{-1}$. 